Active Mathematics In Classrooms: Finding Out Why Children Make Mistakes – And Then Doing Something To Help Them.

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Introduction

This is the third in our series that presents a summary of a workshop from the primary teacher professional learning program that was very successfully conducted in Brunei Darussalam under the supervision of Professor Ken Clements. While it is a mistake to think that you can take a successful program from one culture and expect it to be an instant success in another. Nevertheless, an examination can provide a vehicle for reflection of current classroom practices. For some readers, an examination may provide a welcome revision of some earlier learning or remind them of practices long since rejected and replaced by their current approaches.

The program was called Active Mathematics in Classrooms (AMIC) and had three main groups of teacher participants: workshop writers; workshop leaders; and classroom teachers. A summary of the process is shown diagrammatically below. A full description of the program can be found in Square One (Vol 15, No. 2, pp. 8-15).

The program writers produced nine workshops and the Ministry of Education published a series of booklets to support the workshops and provide the teachers with the necessary theoretical background. The titles of the nine workshops and booklets are:

1. Word problems need not be so difficult
2. Finding out why children make mistakes – And then doing something to help them
3. Developing number sense throughout the primary mathematics curriculum
4. Fractions: Going beyond area models to assist student learning
5. Decimals: Going beyond skill and drill into reality
6. Percentages, linked to decimals and fractions
7. Language issues in upper primary mathematics classrooms
8. Expanding the modes of communication (including technology) in primary mathematics classrooms
9. Teaching mathematics through a problem-solving and problem-posing approach.

The first workshop was presented in our last edition (White, 2005).It is the intention of the writer to now present material from the second workshop (adapted from Salim, 2003). As you can see from the teacher cycle in the figure above, the teachers began by participating in a workshop that covered the material to be used in their classroom.

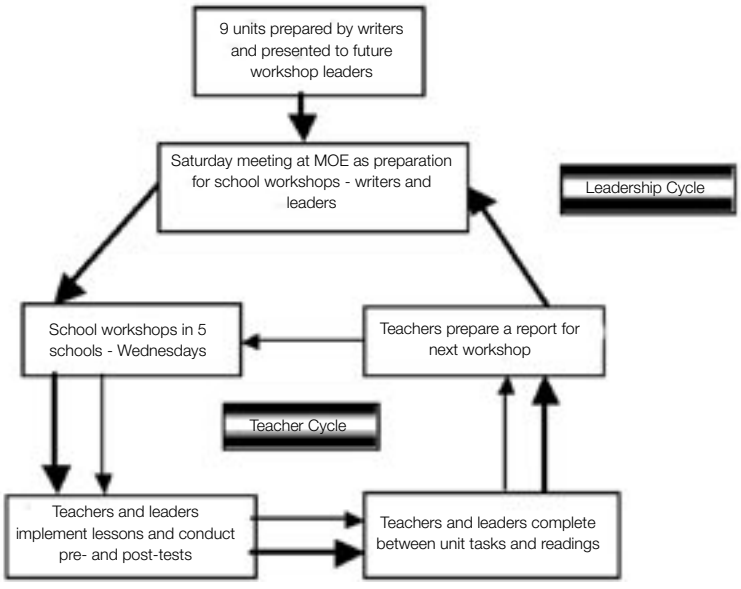

Figure 1. AMIC cycle for writers, leaders and classroom teachers (White & Clements, p. 152 ,2005a).

The Workshop

The workshop has a theoretical foundation based on Newman (1977), who maintained that when a person attempted to answer a standard, written, mathematics question then that person had to be able to pass over a number of successive hurdles: Reading (or Decoding), Comprehension, Transformation, Process Skills, and Encoding. Along the way, it is always possible to make a Careless error. While there are many other theoretical approaches available to teachers, Newman's offers one of the easiest to use and adapt and has proven popular among teachers.

In the past, teaching in the period BC (before constructivism) teachers often had negative feelings about the mistakes that children would make, regarding them as "unfortunate" events that need to be eliminated and possibly avoided at all times. However, the strategy of more drill and practice has been replaced by regarding errors as valuable sources of student thinking. As teachers it is difficult to escape from children's mistakes so it is worthwhile finding out why children make the

mistakes in the first place (and often continue to repeat the mistake). Mistakes can become entrenched, so error analysis is the first step towards doing something relevant which will remove the cause of the mistake.

Although there are many ways to find out why children make mistakes on mathematics tasks, the Newman error analysis/interview procedure is highly regarded.

Workshop Objectives

- To learn the Newman "analysis of errors" strategy for identifying why children make mistakes on written mathematical tasks.
- To use the Newman error analysis interview to find out why real Primary 5 pupils made mistakes on a mathematics test.
- To study the types of errors made by different pupils in a Primary 5 class on some written mathematics tasks, and develop error profiles for the pupils.

Workshop Activities

The teachers were expected to:

- Discuss mistakes which children would be likely to make on some simple paper-and-pencil mathematics questions.
- Administer a pencil-and-paper mathematics test to a Primary 5 class, marks the pupils' scripts, and prepare Newman "interview data" sheets.
- Conduct Newman Interviews with Primary 5 children who attended the workshop for this purpose.

Activity 1: Examination of some common mathematical errors. The teachers discussed the following examples in small groups before joining a large group discussion.

3 + ☐ = 7	A common answer error is 10 "A plus sign means to add"
. 35 +67	A common answer error is 912 "Add the numbers in each column and write the sums under the line
42 -17	a common answer error is 35. "When you subtract, you take the smaller number from the larger".
300 -136	A common answer error is 163 or 174 "You can't subtract a number from zero, so you change zero into nines and the 3 becomes two" "You can't subtract from zero, so you borrow from the 3 and the zeros become tens".
1/2 + 2/3	A common answer error is 3/5 "When you add fractions, you add across the top and across the bottom".
2.06 + 1.3 + 0.38	A common answer error is 2.57. "Line up the numbers and add". This is a form of decimal blindness - they don't see the point.
$5.40 x 0.15	A common answer error is $81.00. "After you do the calculation, bring down the decimal point".

Because teachers are not fortune tellers who can predict how children think all the time, they cannot read what is going on in children's heads. What can they do, then, to find out why their pupils make mistakes on written mathematical tasks? The special interview technique called the "Newman error analysis procedure" provides one useful method for solving the error identification and analysis dilemma.

15 Mathematics Questions: Write the answer to each question in the space provided. The teachers were asked to give the following questions to their sample of year 5 students. How would your study perform on this list? Why not use one of your own recently completed tests and conduct the Newman analysis?

1. Balkis has $4.80 and Said has $6.34. How much more money does Said have than Balkis? Answer:
2. Here are three fractions: 1/3, 1/4, 1/10. Write these fractions in order of size, from smallest to largest.

   Smallest Fraction:_____ Middle Fraction: _____
   Largest Fraction: _____
3. If Raman gives the shopkeeper $5 for a box of flour that costs $3.65, how much change should he get? Answer:
4. Ice cream cost 85 cents each, and apples cost 45 cents each. How much altogether would 7 ice-creams and 5 apples cost? Answer:
5. I had a piece of rope, but someone cut 15 metres off it. When I measured the rope that remained, I found it was only 57 metres long. How long was my original piece of rope? Answer:
6. It takes Kim 15 minutes to walk to school. If school starts at 8:05 a.m., at what time must she leave home so that she can get to school just as school starts? Answer:
7. Radiah is 12 days older than Mary. If Radiah's birthday is January 29, on what day is Mary' s birthday? Answer:
8. A shop is open from 1 o'clock in the afternoon to 5 o'clock in the evening. For how many hours is the shop open? Answer:
9. If the time is now 9 o'clock, what was the time 4 hours ago? Answer:
10. Find the value of 940 - 586. Answer:
11. What time will it be 35 minutes after the time shown on the clock? (10 minutes past 6). Answer:
12. Two exercise books and a pencil cost $2 altogether. If pencil costs 30 cents, how much does an exercise book cost (each book costs the same amount of money as the other)? Answer:
13. There are 12 apples on the table. If you picked up one-third of the apples, how many apples would there still be left on the table? Answer:
14. Pensri works in a shop each day from March 22 to March 26. If she earns $10 a day, how much altogether did she earn? Answer:
15. Bella takes exactly 3 hours to walk 15 km. How long would it take her (if she walks at the same speed) to walk 20 km? Answer:

Conducting The Newman Interview

- Talk to the child in a friendly way, briefly, in an attempt to assist him/her to feel relaxed. Tell him/her your reason for talking to him/her is to help him/her with his/her Mathematics.
- Tell the child you want him/her to do some of the problems that he/she did before, once again.
- Give the child a new test paper and answer sheet, and ask him/her to answer a question for which he originally gave an incorrect answer. Encourage the child to show any working. Say nothing more until he/she is finished.
- Ask the child some or all of the five key Newman questions/requests (see below) for the particular question being considered. Do not help the child at any stage, but note (briefly) any of the child's answers which are especially revealing.
- Decide on the Newman error classification, that is to say the classification that you believed corresponds to where the pupil got off track on the original pencil-and- paper administration of the question (ie., before the interview).
- Repeat 3, 4 and 5 for the next error.

The Five Newman Questions/Requests

1. Please read the question to me.
2. Tell me, what is the question asking you to do?
3. Which method do you use to get your answer?
4. Show me how you get your answer, and "talk aloud" as you do it, so that I can understand how you are thinking.
5. Now, write down your actual answer.

During Step 4, listen carefully to what the child says and decide where you think, when the child originally attempted the question in the whole-class context, the first breakdown point is. Hence classify the mistake/error. (Go at least one step beyond what you think is the first breakdown point)

The Newman Error/Mistake Analysis Procedure

1. To identify reading errors (R): "Read the question to me. If you don't know a word tells me."
2. To identify comprehension errors (C): "Tell me, what the question asked you to do."
3. To identify transformation errors (T): "Now tell me what method you used to find the answer."
4. To identify process skills error (P): "Now go over each step of your working, and tell me what you were thinking."
5. To identify encoding errors (E) - an inability to express an answer in an acceptable form: "Tell me, what is the answer to the question? Point to your answer."

If, when the child attempts the question for the second time, he/she gets the correct answer and, after the teacher has listened to the answers to the Newman requests, the teacher is convinced that the child *originally* made a careless slip, then, the error would be classified as CARELESS (coded as X).

READING ERRORS (coded as R). An error would be classified as READING if the child could not read a key word or symbol in the written problem to the extent that this prevented him/her from proceeding further along an appropriate problem- solving path.

COMPREHENSION ERRORS (coded as C). The child had been able to read all the words in the question, but had not grasped the overall meaning of the words and, therefore, was unable to proceed further along an appropriate problem-solving path.

TRANSFORMATION ERRORS (coded as T). The child had understood what the questions wanted him/her to find out but was unable to identify the operation, or sequence of operations, needed to solve the problem.

PROCESS SKILLS ERRORS (coded as P). The child identified an appropriate operation, or sequence of operations, but did not know the procedures necessary to carry out these operations accurately.

ENCODING ERRORS (coded as E). The child correctly worked out the solution to a problem, but could not express this solution in an acceptable written form.

Between-Unit Activities

After completing a trial run during the workshop, the teachers were expected to complete the following before the next workshop and perform each of the following activities with their class:

- Administer the pencil-and-paper test to the class;
- Analyse errors made by some pupils using the Newman interview technique;
- Fill in data sheets indicating pupils' Newman error categories.

1. T*eacher's Evaluation and Mark Sheets:* Workshop participants were expected to fill in the mark sheets provided as well as an evaluation form. These were submitted at the next workshop, after each participant had conducted the Newman interviews with at least two pupils.
2. *Prepare a Report for the Next Workshop:* Each workshop participant was expected to give a 5-minutes report on his/her findings at the next workshop. They reported to workshop participants what they observed when performing the activities with their class and were asked to comment on the advantages and disadvantages of the activity.
3. *Between-Unit Reading:* They were also expected to read the following papers
    a. Anthony, G. (1998). It's all right to be wrong. *The Australian Mathematics Teacher,* 54(4), 34-37
    b. Clements, M. A. (2004). *Analysing Errors Made by Pupils on Written Mathematics Tasks.* Sultan Hassanal Bolkiah Institute of Education, Universiti Brunei Darussalam
    c. Marinas, B., & Clements, M. A. (1990). Understanding the problem: A prerequisite to problem solving in mathematics. *Journal of Science and Mathematics Education in South*

East Asia, 13(1), 14-20

d. Smith, R. (1989). What's going on in their Heads? *Mathematics in School,* 18(5),33.

The only reading that would be difficult to obtain is the second one and so what follows is a very brief summary of the key points of this reading. In this paper different types of interviews that have been used successfully in education research were considered. The main example of a structured interview approach discussed was the Newman (1977) diagnostic technique, a variant of which is the technique developed by Casey (1978). The paper also looked at data arising from teaching experience interviews (as used in the Erlwanger (1975) study, and used by interviewers in constructivist-oriented research- (see, for example, Wright, 1990). This summary will concentrate on the Newman diagnostic interviews.

I will not represent the Newman material already listed above: the process and questions. From a research methodology perspective, it is important to realise that these key questions were carefully linked to M. Anne Newman's theory on why students make mistakes on written mathematical tasks (see Newman (1977) and Newman (1983) for discussion of her theory, and of her research based on that theory).

"Newman also allowed for "careless errors" and errors due to a lack of motivation on the part of students (see Newman, 1977, 1983). Newman error analysis research by Marinas and Clements (1990), Singhatat (1991), and Clements and Ellerton (1992), in Southeast Asia reported that about 70% of errors made by students on standard word problems could be attributed to a lack of comprehension or to an inability to select an appropriate sequence of operations (that is, in Newman's terms, to carry out the required "Transformation"). The strength of such data suggests that one of the urgent agendas of mathematics education research is to establish a way of addressing this state of affairs. Above all else, Newman research points to the students' lack of a deep understanding of mathematical vocabulary, semantic structure, and the absence of links between the students' formal language and mathematical skills, and their personal worlds. In other words, the Newman research procedure provides a framework for investigating each of the major themes discussed above (Clements & Ellerton, 1996). Clements (1999) argued that mathematical modelling should be present whenever a learner attempts to solve a real-life problem or a mathematics word problem. In that sense, Newman's transformation often involves mathematical modelling. Often, this process of transformation or mathematical modelling is not evident in mathematics classroom discourses." (Clements, 2004, pp. 2-3).

Professor Clements also provided some advice for inexperienced users. " From my long experience of observing interviewers attempting to follow Newman procedures, I can say that inexperienced interviewers often find it difficult to stop themselves from helping students to answer the questions. Then, when a student gets a correct answer (as a result of the help given by the interviewer) the error is (often incorrectly) classified as a "Careless error." In Newman interviews it is important that the interviewer does *not* help students over a hurdle. Usually, in Newman interviews, the interviewer is wise to go one step beyond the first hurdle the interviewee misses for any question. That is because sometimes a student knows how to do something but is unable to express that knowledge verbally in an interview." (Clements, 2003, pp 4-5).

And further he states "In Newman research it is normal for students to have answered the written question *twice* before being asked the sequence of Newman questions for that test question. The first time usually occurs in a whole-class test situation. The Newman interview should take place as soon as possible after that first test, and steps should be taken to try to ensure that students who took the test do not talk to each other about the solutions they gave (or their teacher does not talk to the class about the test, or any question on the test). The second time the student attempts the written question is usually in the interview situation, *before* the Newman questions are asked.

When the interviewer begins to pose the Newman questions to a student, the interviewer should already have two pieces of data concerning the student's interaction with the question. The first datum is the answer given in the whole-class test situation; the second is the answer given when the child attempted the question for a second time. If the test is such that "student working" is shown, then this working provides extra data. Responses given to the questions asked in the Newman interview provide important additional data. Note that the aim of the whole exercise is to decide, by considering all the data available, why the student made the mistakes he/she made on the original test questions (which were questions on the test administered in the whole-class context).

The decision on why a student made a mistake on a particular test question is usually made by the interviewer immediately after the interviewer has asked the student the Newman questions in relation to that question. The interviewer should have a coding sheet on which the student's original answer was shown. The interviewer should enter on the coding sheet the second answer given by the student (before the Newman questions were asked). The coding sheet should also provide space for the interviewer to write a code (*R, C, T, P, E, or X*) indicating the decision made on the Newman error category (on the original whole-class test) that the interviewer deemed to be appropriate given all the data now available. The coding sheet should also have some space available if the interviewer wants to make a brief comment on any matter arising in the Newman interview.

After the interview is complete, the following data will be available so far as an error made by the student when attempting the original (whole-class) test: the original (whole-class test) answer given to the question;' the

second answer given to the question; the Newman classification made by the interviewer as a result of student responses to the Newman questions." (Clements, 2003, p.5).

Regarding the issue of careless errors, Professor Clements states (2003, pp. 5-6). " In Newman research a careless error has been defined as one which occurred even though the student knew (from a cognitive perspective) exactly how to gain a correct answer to the question at the time the incorrect answer was given and would be expected to give the correct answer when responding to the same question at some later time.

Thus, if a student gave an incorrect response in the original whole-class test situation but then gave a correct answer immediately before the Newman interviews, then the interviewer would suspect that an X (Careless error) classification of the error might be appropriate. Data from the Newman interviews should then enable the interviewer to decide whether the X-classification is really appropriate. If, during the Newman interview for that question, it becomes clear that the student was not sure which of the two answers that he had given - the incorrect one, given when the test was administered to the whole class, or the correct one, given just before the Newman interview took place - was correct, then the original error should not be classified as X (Careless)".

The conclusion of the article discusses the importance of producing summary sheets and how to best extract patterns of errors that can provide direction for future classroom planning. The two modifications are also discussed.

While this is a far from satisfactory summary, if the reader is keen then the main ideas can be found in the list of papers by Clements in the reference section. The reference list provides a good foundation for further reading. Alternatively, if you contact me at al.white@uws.edu.au, I will send you a copy of Clements' 2003 paper.

Conclusion

In this attempt to condense another of the workshops, it is hoped that enough material was provided to do justice to the original. While there are many ways for finding out why children make mistakes, the approach presented in this unit has been popular with teachers. It offers an easily understood framework, backed by research, for working with student errors. So how did your students do?

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